

Some studies related to Fermat's Last Theorem

John van der Linden

September 29, 2026

Corrected and otherwise revised October 15, 2026

SETUP AND MINIMUM VALUES

[0] $a^n + b^n = c^n$ where $a < b < c$ and a , b , c , and n are positive integers with $n \geq 2$

since $c > a$ and $c > b$, c can be written as $x+a$ or $y+b$ where x and y are positive integers

[1] thus $a^n + b^n = (x+a)^n = (y+b)^n$

now expand $(x+a)^n$ and $(y+b)^n$:

$(x+a)^n = x^n + a^n + \text{remaining terms (call their sum "f")}$

$(y+b)^n = y^n + b^n + \text{remaining terms (call their sum "g")}$

thus $a^n + b^n = x^n + a^n + f = y^n + b^n + g$

subtract from both sides

$b^n = x^n + f$

$a^n = y^n + g$

f and g are both positive integers, thus $b^n > x^n$ and $a^n > y^n$

take n th root of both sides

$b > x$ and $a > y$

write b as $x+s$ and a as $y+r$ where s and r are positive integers

now back to [1]

$a^n + b^n = (x+a)^n = (y+b)^n$

$a^n + b^n = (x+(y+r))^n = (y+(x+s))^n$

$a^n + b^n = (x+y+r)^n = (x+y+s)^n$

thus $r = s$; rename this integer Z .

[2] then $a^n + b^n = (x+y+Z)^n$

by definition, $a \neq b$ and since $c = x+a = y+b$, $x \neq y$

[3] $y+Z = a$ and $y < a$, thus **a must be at least 2** (if $y = Z = 1$)

$b > a$ so **b must be at least 3**

[4] thus $a + b \geq 5$

now back to [2]:

$a^n + b^n = (x+y+Z)^n$

recall that $b = x+Z$ and $a = y+Z$. thus $x = b-Z$ and $y = a-Z$. Substitute these values:

$a^n + b^n = ((b-Z)+(a-Z)+Z)^n$

[5] $a^n + b^n = (a + b - 2Z + Z)^n = (a + b - Z)^n$

Suppose we imagine that $(a, b, Z) = (2, 3, 1)$ works for some unknown integer value of n . Substituting these values of a , b , and Z into [5] gives

$$[6] \quad 2^n + 3^n = (2 + 3 - 1)^n = 4^n$$

Divide both sides by 4^n

$$(1/2)^n + (3/4)^n = 1$$

Both terms on the left are less than 1 and get smaller as n increases, thus n must be small. Substituting $n = 1$ gives $(1/2) + (3/4) = 1.25$ which is greater than 1

Substituting $n = 2$ gives $(1/4) + (9/16) = 0.8125$ which is less than 1

All larger values of n will give a value for $[6]$ that is less than 0.8125, since both exponent terms are decreasing

Thus n must be between 1 and 2 if $a = 2$, $b = 3$, and $Z = 1$. But this violates our initial condition that n is a positive integer

[7] Thus $a = 2$, $b = 3$, and $Z = 1$ cannot be correct

Is Z wrong?

Recall that $b = x+Z$ and $a = y+Z$, where $c = x+a = y+b$. We know $1 \leq x$ and $1 \leq y$, so $1 \leq b - Z$ and $1 \leq a - Z$. Thus $Z \leq a - 1$ and $Z \leq b - 1$. If our current values $a = 2$ and $b = 3$ have a solution then

$$Z \leq a - 1$$

$$Z \leq 2 - 1$$

$$Z \leq 1$$

So there are no other possible values of Z for $a = 2$ and $b = 3$, and the only one available, $Z = 1$, leads to a value of n that violates initial conditions. Thus either $a = 2$, $b = 3$, or both cannot satisfy $[0]$, which means that either $a \geq 3$, $b \geq 4$, or both must be true. Since the minimum value of either a , b , or both increases by 1 regardless, we know that $a + b \geq 2 + 3 + 1$ or **$a + b \geq 6$** .

Suppose $a + b = 6$. Then we have

$$a^n + b^n = (a + b - Z)^n$$

$$a^n + (6 - a)^n = (6 - Z)^n$$

We know that $Z \leq a - 1$, so $(6 - Z)^n \geq (6 - (a - 1))^n$. Thus

$$a^n + (6 - a)^n \geq (7 - a)^n$$

Suppose we assume $a = 2$ (still our current minimum value for a). Then

$$2^n + (6 - 2)^n \geq (7 - 2)^n$$

$$2^n + 4^n \geq 5^n$$

Divide by 5^n

$$(2/5)^n + (4/5)^n \geq 1$$

Since both bases on the left hand side are fractions < 1 , the largest possible value for the left hand side (with $n \geq 2$) will be when $n = 2$. Testing with this value gives

$$(4/25) + (16/25) = 20/25 \geq 1$$

So $a = 2$ cannot be a solution. Now try $a = 3$.

$$(a/(7-a))^n + ((6-a)/(7-a))^n \geq 1$$

$$(3/4)^n + (3/4)^n \geq 1$$

$$2 \cdot (3/4)^n \geq 1$$

$$(3/4)^n \geq 0.5$$

Again, the largest possible value for the left hand side (with $n \geq 2$) will be when $n = 2$. Testing with this value gives

$$(9/16) \geq 0.5, \text{ so the function evaluates correctly when } n = 2. \text{ Now try } n = 3.$$

$(27/64) \geq 0.5$, so the function evaluates incorrectly when $n = 3$. All larger values of n will lead to even smaller values on the left hand side, so only $n = 2$ works when $a = 3$.

However, if $a = 3$ when $a + b = 6$, then $b = 3$. But we know $b > a$. And if $a > 3$, then b must be less than 3 for $a + b$ to equal 6, which also violates the initial condition that $b > a$. Thus there are no workable values of a , and **$a + b = 6$ cannot be a solution to [0].**

[8] Therefore, **$a + b > 6$ or $a + b \geq 7$, and $a \geq 3$ and $b \geq 4$.**

While we're establishing minimum values for variables: note that $c = x + a = b + y$, and because $b > a$, it follows that $x > y$. We also know $y \geq 1$, so

$$x > y \geq 1$$

$$[9] \quad x \geq 2$$

What about Z ? For the Pythagorean solution $3^2 + 4^2 = 5^2$, $Z = (3+4) - 5 = 2$. Can Z ever be smaller than this? That is, can Z ever equal 1?

From [5], we have:

$$a^n + b^n = (a + b - Z)^n$$

$$(a^n + b^n)^{1/n} = a + b - Z$$

The function $f(n) = (a^n + b^n)^{1/n}$ is strictly decreasing for $n \geq 2$, so

$$(a^2 + b^2)^{1/2} \geq (a^n + b^n)^{1/n}$$

$$[10] \quad (a^2 + b^2)^{1/2} \geq a + b - Z$$

Suppose $Z = 1$. Then we can write

$$(a^2 + b^2)^{1/2} \geq a + b - 1$$

Square both sides

$$a^2 + b^2 \geq (a + b - 1)^2$$

$$a^2 + b^2 \geq (a + b)^2 - 2(a + b) + 1$$

$$a^2 + b^2 \geq a^2 + 2ab + b^2 - 2(a + b) + 1$$

$$2a + 2b \geq 2ab + 1$$

$$2a + 2b - 1 \geq 2ab$$

Divide by $2ab$

$$[11] \quad 1/b + 1/a - 1/(2ab) \geq 1$$

For $a \geq 3$ and $b \geq 4$, the two leftmost terms must be less than $1/4 + 1/3 = 7/12$, which is less than 1.

Subtracting the final $1/(2ab)$ term will take the left side even further below 1.

As a result, [11] has no solutions for $a \geq 3$ and $b \geq 4$, meaning the supposition $Z = 1$ must be false.

[12] **Therefore, Z must be greater than or equal to 2.**

Can $Z = 2$ be true for any values of a and b other than $a = 3$ and $b = 4$?

Substitute $Z = 2$ into [10]:

$$(a^2 + b^2)^{1/2} \geq a + b - 2$$

Square both sides

$$a^2 + b^2 \geq (a + b - 2)^2$$

$$a^2 + b^2 \geq (a + b)^2 - 2 \cdot 2(a + b) + 4$$

$$a^2 + b^2 \geq a^2 + 2ab + b^2 - 4(a + b) + 4$$

$$4(a + b) \geq 2ab + 4$$

$$2a + 2b \geq ab + 2$$

$$2a + 2b - 2 \geq ab$$

Divide by ab

$$[13] \quad \frac{2}{b} + \frac{2}{a} - \frac{2}{ab} \geq 1$$

We know this holds for $a = 3$ and $b = 4$. What about for larger values of a and b ?

Try $a = 3$ and $b = 5$

$$\frac{2}{5} + \frac{2}{3} - \frac{2}{15} \geq 1$$

$$\frac{6}{15} + \frac{10}{15} - \frac{2}{15} \geq 1$$

$$[14] \quad \frac{14}{15} \geq 1$$

The above inequality is false, so $a = 3$ and $b = 5$ cannot be a solution when $Z = 2$.

Try $a = 4$ and $b = 5$

$$\frac{2}{5} + \frac{2}{4} - \frac{2}{20} \geq 1$$

$$[15] \quad \frac{8}{20} + \frac{10}{20} - \frac{2}{20} \geq 1$$

$$[16] \quad \frac{16}{20} \geq 1$$

Again, the inequality is false, so $a = 4$ and $b = 5$ cannot be a solution when $Z = 2$. In [15], note that the leftmost two terms sum to $\frac{18}{20}$, which is just less than 1. As a and b increase, these two terms will shrink, so their sum will also shrink, declining further below 1. Subtracting the final $\frac{2}{ab}$ term will push the left side further still below 1. This means that $a > 4$ and $b > 5$ cannot be a solution when $Z = 2$.

[17] As a result, **$a = 3$ and $b = 4$ are the only possible values of a and b when $Z = 2$.**

[18] It follows that **$Z > 2$ when $a + b > 7$.**

FINDINGS ABOUT THE SYSTEM

$$a^n + b^n = c^n = (x + a)^n = (y + b)^n = (a + b - Z)^n = (x + y + Z)^n = (x + Z)^n + (y + Z)^n$$

$$c = a + b - Z = x + y + Z$$

$$x + Z = c - y$$

$$a - Z = c - b$$

$$a \geq 3$$

$$b \geq 4$$

$$x \geq 2$$

$$y \geq 1$$

$$Z \geq 2$$

$$a = y + Z$$

$$b = x + Z$$

$$x = b - Z$$

$$y = a - Z$$

$$b > x$$

$$a > y$$

$$b > a$$

$$b > y \text{ (follows from previous two)}$$

$$c < b \cdot \sqrt{2} \quad (\text{p. 116})$$

$$y / b < \sqrt{2} - 1 \quad (\text{p. 116})$$

$$\ln(1 + y/b) < \ln(2) / n$$

$$x > y, \text{ because } b > a \text{ and thus } x + Z > y + Z$$

$$a + b \geq 7$$

$$ab \geq 12$$

$$x + y \geq 3$$

$$x + y = a + b - 2Z \text{ so } x + y + 2Z = a + b \text{ and } x + y < a + b$$

$$x + y < a + b$$

$$3 \leq a + b - 2Z$$

$$9 - (a+b) \leq Z \leq a + b - 5$$

$$b - a = x - y$$

$$b - x = a - y = Z$$

$$x + a = b + y$$

$$(x + a) = (x + y + a + b) / 2 = (y + b) = c$$

$$x + a = c$$

$$y + b = c$$

$$\text{(the above two because it's defined so at the very beginning of the proof; but also, } x + a = b + y \text{ and } c = (a + b + x + y) / 2)$$

$$x + a \geq 5$$

$$b + y \geq 5$$

$$x + a > 2y$$

$$x + a < 2b$$

$$x < 2b - a$$

$$c = (a + b + x + y) / 2$$

$$c + Z = a + b$$

$$c - Z = x + y$$

$$Z = a + b - c = c - x - y$$

$$Z = (a + b - x - y) / 2$$

$$z - 1 \geq (a + b - c) / 2 \geq 1$$

$$z \geq (a + b - c + 2) / 2 \geq 2$$

$$b \leq (a^n + b^n)^{1/n} \leq b^{\sqrt[2]{2}} \quad (\text{p. 116})$$

$$b \leq (a^n + b^n)^{1/n} \leq ((a + b)^n - 2^* a^* b)^{1/n} \quad (\text{p. 116})$$

$$(a^n + b^n)^{1/n} \geq (3^n + b^n)^{1/n} \geq (9 + b^n)^{1/n}$$

$x + y + 2 \leq (a^n + b^n)^{1/n} \leq a + b - 2$ (all values of the center term for $n=2$ seem to be exactly halfway btwn the upper and lower bounds)

$(x - b + 5)^n \leq a^n + b^n \leq (x + 2a + b - 5)^n$ (p. 104 top) (like the the originally found bounds for Z , not super helpful - lower bound often negative, upper bound enormous)

More inequalities or equivalences (p. 108):

$(x + y - 2)^n$	$\leq a^n + b^n \leq$	$(a + b + 2)^n$	
$(x + y - Z)^n$	$\leq a^n + b^n \leq$	$(a + b + Z)^n$	
$(c - Z)^n$	$\leq a^n + b^n \leq$	$(c + Z)^n$	
$((a+b+x+y)/2 - Z)^n$	$\leq a^n + b^n \leq$	$((a+b+x+y)/2 + Z)^n$	
$(x + a - Z)^n$	$\leq a^n + b^n \leq$	$(x + a + Z)^n$	but $x + a = c$
$(y + b - Z)^n$	$\leq a^n + b^n \leq$	$(y + b + Z)^n$	but $y + b = c$
or $(P - Z)^n$	$\leq a^n + b^n \leq$	$(P + Z)^n$	
$(x + a - Z)^n$	$\leq (x + a)^n \leq$	$(x + a + Z)^n$	
$(y + b - Z)^n$	$\leq (y + b)^n \leq$	$(y + b + Z)^n$	
$(x + y)^n$	$\leq a^n + b^n \leq$	$(a + b)^n$	
$(x + y)^n$	$\leq (x + a)^n \leq$	$(a + b)^n$	
$(x + y)^n$	$\leq (y + b)^n \leq$	$(a + b)^n$	
$(x + a)^n$	$= ((x+y+a+b)/2)^n =$	$(y + b)^n$	

or simply $(x + a) = (x+y+a+b)/2 = (y + b)$

Also note that, at least for all Pythagorean triples I've examined, $a + b$ and $x + y$ appear to be equidistant from c . The distance is equal to Z , because $a + b - Z = x + y + Z = c$.

Some variable values when $n = 2$

a	b	c	x	y	Z	a^2	b^2	c^2
3	4	5	2	1	2	9	16	25
5	12	13	8	1	4	25	144	169
8	15	17	9	2	6	64	225	289
7	24	25	18	1	6	49	576	625

DISCUSSION

In addition to the above, I have (as of 09/29/26) about 100 notebook pages of calculations involving this system of variables, including several unsuccessful attempts to prove the theorem with simple algebra. In those pages, I worked out the variable equations/identities in the "Findings about the system" section, above, wherein I have sometimes referenced the corresponding notebook page numbers.

An AI agent was helpful in gathering information and shaping my understanding of some of the math used in this project, but the core ideas and directions are my own (though they have likely also been pursued previously by others, whose work I have not yet explored).